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THE METHOD OF THE STUDENTS' COMPETENCE RATING: KNOWLEDGE, ABILITIES, SKILLS AND PERSONAL CHARACTERISTICS

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ABSTRACT

This paper describes the new fuzzy membership complex function sets which are called status functions. It demonstrates how these function sets might be used in the mathematical description of the multidimensional rating of competences during education process of individuals and its ability to increase professional level, viewed both from external social role, and from internal readiness for the change. The method for estimation of effectiveness of education process is also proposed.

To estimate the effectiveness of education process we propose to use transfer function or frequency response function which can be calculated as discrete Fourier transform at the interval of rating variable.

JEL CLASSIFICATION

■ C02 ■ I21 ■ Status Functions ■ Rating ■ Competences ■ Fuzzy ■ Educatiton

INTRODUCTION

Nowadays, some methods which were originally developed for the analysis of the fundamental physical phenomena have been applied successfully to the solution of various problems arising in social systems [1,2]. The use of statistical approaches and conception of probability description of characteristics of social processes leads to appearance of corresponding models and equations which can be investigated via computer simulation and different information technology methods. After fundamental works on fuzzy sets theory [3] the so-called linguistic probabilities became common and widely used instrument to the solution of problems of intellectual management. The important problem is a formulation of models for the description of principles of intellectual activity of a person [4]. Many objects which are connected with the personal characteristics can be considered as dynamic systems. Such objects consist of many parts cooperating with each other with more or less complexity. Additionally, synergetic methods can be applied to study the cognitive activity of human brain, which is one of the most complicated systems [5].

On the other hand, it is necessary to mention the interest to introduce the mathematical methods and rating scales for research of psychology of individual. For example, in Ref. the results of study of correlation among mental abilities of individual and general knowledge are presented. The correlations of aspects of personal characteristics were calculated. Meta-analytical researches show that there are fundamental feelings of special measures of perception, affect, and ability to strong-willed movement are exist [7].It leads to rather small set of parameters which are connected each other and influence intellectual development different manner.

For the solution of a management problem of the professional knowledge formation process fuzzy set approaches were used successfully [8,9,10]. This seems perspective and well-established technique, because the use of fuzzy sets rules allows introducing measuring scales and accurately formulating the rules for control actions. Let us notice that the application of fuzzy sets method has a few of lacks. At first, large amount of information contains difficult for formalization intuitive preferences of the person which forms lists of rating parameters and formulates membership functions. At second, range of fuzzy estimations is static.

In present work, it is proposed to use new membership functions which are named as status functions in the mathematical description of the multidimensional rating of competences during education process of individuals and its ability to increase professional level, viewed both from external social role, and from internal readiness for the change. Introduction of status functions will allow overcoming the specified lacks of applicability fuzzy set theory.

Problem formulation

For modeling and forecasting of results of process of training/education we will consider that except measurable knowledge, abilities, skills (which we will denotes as general knowledge (GK) the result of training is influenced by personal characteristics: GK & Personality. For measurements (rating) of GK fuzzy set method can be successfully applied. These models are similar to traditional rating. The rating is presented as average value of some variable which is the set of ratings of components included into it. Rating of GK may be given using fuzzy and linguistic variables [11,12]. These approaches have some advantages in comparison with others systems (for example, probability systems). It gives possibilities to obtain not only values of ratings, but also their degree of reliability and distribution [13].Then the traditional rating can be presented as a expected value of distribution of a convex combination of fuzzy sets received as a result of rating during classes, for example.

Membership function of resulting distribution actually depends on time and some space-like rating "coordinates". In traditional fussy-logic rating of GK positive-determined membership functions (MF) are used which are overlapped plotted vs. these space-like coordinates. The resulting MF (RMF) is the sum or product of weighted partial MF, which were derived from experts ratings. The RMF in time "moves" back and forth along its "coordinates".

Measurement of personal part of competence can not be done using instantaneous tests, polls and similar ratings. The effect of personal characteristics, such as the content-value-relations, emotionally-strong-willed regulation and mobilization readiness of realization of the corresponding competence can be shown only in actions, influencing on GK getting. Thus we have something like probability distribution function which moves in time according external "force" and internal "intents" not seen to observer. The analogy exists: quantum mechanical particle is characterized by probability distribution of finding

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it in given points of space, and the momentum of particle determines its further movement.

To utilize this analogy we should construct the analog of wave-function and momentum. We shall attribute the momentum value to personal characteristics of trainee, and square modulus of wave-function to RMF.

To do that we introduce orthogonal set of basic complex valued MF, and phase part of these MF will reflect the personal characteristic of trainee. The amplitude of RMF will be obtained as weighted superposition of basic MF with corresponding phases. Using the sociologists terminology, we will call them as status functions (SF).

Let we suppose, that there exists some function describing the behavior of individual. We can measure it in various experiments, for example, make rating during of training process. GK is the part of this function. Another part (Personality) can not be measured in the same experiment in which we measure GK at given moment. It only contributes to its future SK result. This function in a certain sense is similar to wave function of quantum particles. This function for free quantum particles is:

$$\psi(r) = A(r)e^{i2\pi kr}, \quad (1)$$

where r is the coordinate in some space, k is the characteristic connected with particle momentum (which corresponds to Personality), $A(r)$ is the amplitude (which corresponds to General Knowledge). Energy of a particle is proportional k^2.

We believe this function can describe external social value of person (GK) and possibility of its change under the influence of personal characteristics (Personality). We will call it "status function".

For measurement of momentum it is necessary to measure particle coordinates in two successive time moments, calculation of velocity and to multiple to inertness measure (particle mass).

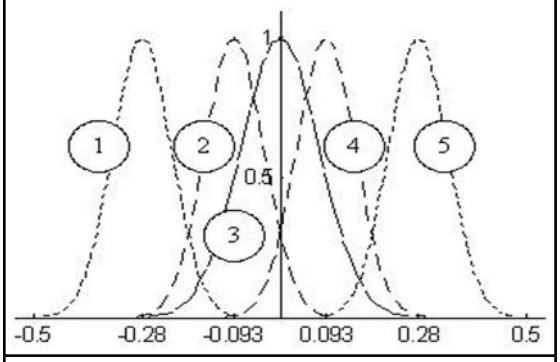
Therefore, the problem of modeling and forecasting of results of process of training is proposed to be solved by using the complex status function. This function may be used for characterization of current status of the system investigated and prediction of its dynamics. Instead of positively defined fuzzy membership functions, we will design complex wave functions and we use them for characterization of possible states of trainee and education process.

The paper is organized as follows. In Section 3 the orthogonal set of SF is introduced and discussed. In Section 4 the experimental results of competence rating with use the SFs during the course "Matrix Optics" for graduate students are presented. In Section 5, some properties of "density of probability" of RSF are discussed in Section 6. Concluding remarks and directions of further investigations are presented in conclusions to this paper.

Membership functions design

Let us illustrate the described approach for competence rating. We suppose that the rating grades are: informative, practical, reproductive, productive, research. Rating variable is *r*. Corresponding to that grades Gaussian MFs are shown in Fig.1a and they are determined at the interval [-0.5; 0.5]. Membership functions have been chosen being unity at maximum, symmetric against *r=0* and being sufficiently small at interval edges (< 10^{-7}). The mean values of MFs also lies in an interval [-0.28; 0.28]. The rating grades in linguistic terms are: -0.28 – informative,-0.093 – practical, 0. – reproductive, 0.093 – productive, 0.28 – research (Fig. 1a). These MFs are common for fuzzy methods. Thus the rating grade is calculated as mean value *r* on resulting membership function.

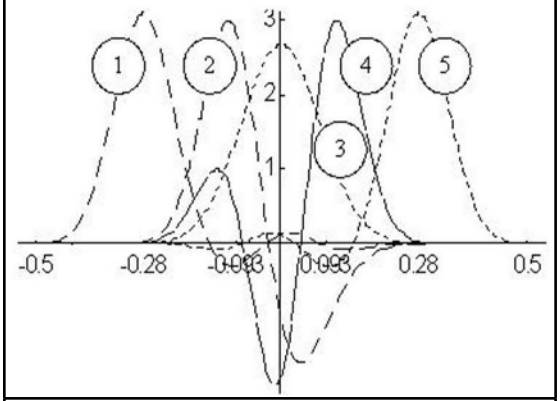
Figure 1a: Membership functions of GK part of the competence before orthogonalization



Note: . The corresponding of membership function is 1 – Informative, 2 – Practical, 3 – Reproductive, 4 – Productive, 5 – Research

Source: own results

Figure 1b: Membership functions of GK part of the competence after orthogonalization



Note: . The corresponding of membership function is 1 – Informative, 2 – Practical, 3 – Reproductive, 4 – Productive, 5 – Research

Source: own results

The linguistic statements (for example, «training is effective», or «the person has prescribed competence and positively treated for further training» and so on) can be derived by some procedures from resulting MF, based on expert rating. These statements allow to make management decision. Using this approach mainly the mean value can be obtained. However, the new status function can be used to obtain the more detailed information about trainee and education process.

For the complex "membership function" design let us first carry out the orthogonalization of basic set using the algorithm of Gram-Shmidt. It gives us possibility to use something like a superposition principle in the procedure of derivation of resulting SF. To obtain the rating grade of competence the direct "measurement" is not possible actually. The rating grade should be received using indirect experiments. It is the result of interaction between examinee and trainee during the direct contact during oral examination or using "measuring" tests. In general we find probability

$$|\psi|^2 = \psi^*\psi \quad (2)$$

of correspondence of measured characteristics to the predefined ones: here ψ^* is complex conjugate to ψ. Thus, during rating process of results of education we "measure" quantity which is similar to probability density. It is commonly accepted that the probability density should be normalized to unity.

Orthonormal basis allows the use of superposition principle. We will characterize a state of trainee as the sum on states in which it can stay, with coefficients being measured via expert rating or test results. This is similar to quantum mechanics description of behaviour of quantum particle(s) via state vector. In our case we should define phase part of our SF as "state vector".

The complex part of SF can not be measured in one experiment similar to situation in quantum mechanics. However it gives a contribution to result, like a phase of the wave or wave packet. Using quantum mechanical analogy we can propose to introduce the phasor similar to plane wave: exp(*ikr*). For representation of the personal characteristic of trainee we have introduced three rating levels (grades). For the lowest level of k = -1 we suppose to apply linguistic term "passive" which means inert, inactive and indifferent to the surroundings. Usually, persons with the underscored self-evaluation are passive, and this position is certainly defeated.

The following level occurs at k = 0. We term it as "declarative". It is the level corresponding to style of behavior «not to be, but to seem». In this case, the person shows himself demonstrating declaration of the possibilities for creation of the reputation but not for achievement of the goals and not for self-improvement.

The highest level in this grade system is k = 1. This is "active" level, corresponding to active position. The term "activity" is widely used in various areas of a science. In this article we understand "activity" as personal characteristic which is demonstrated as inner readiness for interaction with environment. It is based on requirements and interests of the personality, and is characterized by aspiration and desire to operate, commitment and persistence, vigor and an initiative.

Let us construct basic functions as product of functions depicted in Fig.1b and e^{ikr}: $\psi_{lk}(r) = f_l(r)\exp(i2\pi kr)$. The number of basic functions is 5 × 3 =15. We can find "momentum" grade in spirit of quantum mechanics:

$$\int_{-0.5}^{0.5} dr\, f_l(r)\exp(-i2\pi k_l r)\left[-i\frac{\partial}{2\pi\partial r}\right]\left[f_l(r)\exp(i2\pi k_l r)\right] = k_l. \quad (3)$$

Average value of rating coordinate doesn't depend from k ("momentum") in these basic states. These new function remain orthogonal. Superposition of basic functions (elementary SF) leads to SF which depends on moments and GK.

We have obtained the functions which are differing from MF of fuzzy sets method. These functions are similar to wave functions from quantum mechanics. We named them as elementary status functions and its superposition will be resulting status function.

The experiment set up

Further, we demonstrate the application of status functions for investigation of education process and competence rating of graduate students' class on «Matrix optics».

Table 1: Example of rating list

Competence identifier	The content of competence		
		GK	Personality
…			
For student #2			
CIn.1	Electromagnetic theory of light fundamentals	Practical	Declarative
CIn.2	The equation for plane wave	Reproductive	Active
CIn.3	Concept of a monochromatic wave	Research	Active
…			
COut.1	Derivation of Fresnel formulae for normal incidence	Reproductive	Declarative
COut.2	Light scattering as excitation and radiation processes	Practical	Active
COut.3	Physical sense of medium polarization	Practical	Active
…			
For student #3			
…			

Source: Own research

First we list the input and output competences. Part of rating list for different students is present in Table 1 in more detail.

Let us calculate the weight factors for each competence in the resulting rating. For calculation of correlations the technique from Ref. 14 is used. The weight factors are defined by correlation coefficient of j-th competence with a total sum of points(grades):

$$r_{jy} = \frac{SP_{jy}}{\sqrt{SS_j \cdot SS_y}} \quad (4)$$

where $SS_j = \sum_{i=1}^{n} K_{ij}^2 - \frac{(\sum_{i=1}^{n} K_{ij})^2}{n}$, $SS_y = \sum_{i=1}^{n} Y_i^2 - \frac{(\sum_{i=1}^{n} Y_i)^2}{n}$,

and $SP_{jy} = \sum_{i=1}^{n} K_{ij}Y_i - \frac{(\sum_{i=1}^{n} K_{ij})\cdot(\sum_{i=1}^{n} Y_i)}{n}$,

K_{ij} is the number of points of i-th student for j-th competence, n is a number of students, Y_i is total number of points of i-th student.

Large values of r_{jy}, describes larger contribution of j-th competence to total grade. The competence weight is defined as the ratio of its correlation factor to the sum of all factors.

2. The status function for the each student is:

$$\psi_i(r) = \sum_{j=1}^{m} w^j f_i^j(r)\exp(i2\pi k_i^j r). \quad (5)$$

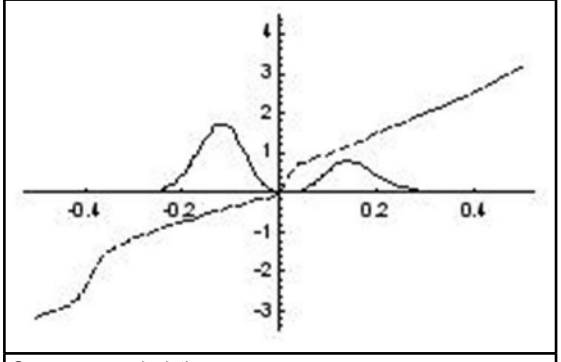
Here w^j is weight factors (GK); f_i^j are function from (4); m is the total number of competences in experiment.

Interpretation of experimental results

The status function contains full information about student which take part in rating process. Some examples of status functions are present in Figs.2 and 3. First one was given for participant #2 (i=2). In this case we observe a high level of average value of GK and Personality activity. We can see positive slopes of the argument at the maxima of $|\psi_i(r)|^2$, corresponding to the wave propagating along r-axis. We see also the positive variations (increase of average grade) after class.

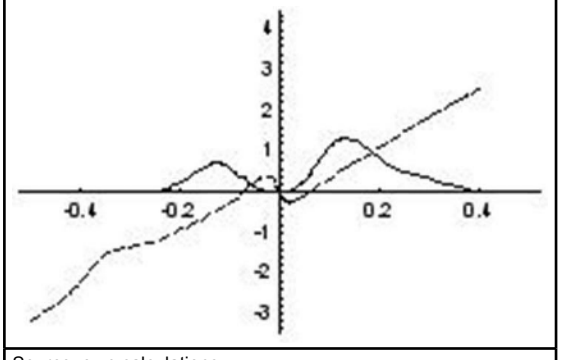
The next example is the student #8 (Fig.3). This participant has good knowledge and high grade initially. He is self-assured, lazy and passive.

Figure 2a: The status function of competence set for i=2. Solid curve is for modulus, dashing curve is for argument input competence set.



Source: own calculations

Figure 2b: The status function of competence set for i=2. Solid curve is for modulus, dashing curve is for argument output competence set.



Source: own calculations

The results of education impact are not so impressive. Mean value of square of modulus of status function represent the average grade.

$$m = \int_{-} r\,|\psi(r)|^2\,dr \qquad (6)$$

These values are present in Table 2.

Table 2: Examples of changes of status function's characteristics

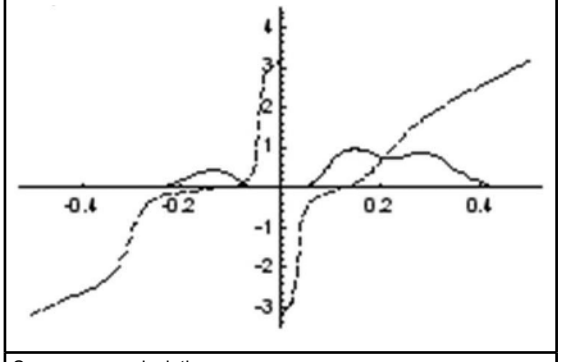
student	Characteristic	Input	Output
i=2	Mean value	-0,00859493	0,0286484
	Dispersion	0,00623844	0,00972143
	Asymmetry	2,09705E-05	0,00169325
	Excess	0,000206335	0,000562598
i=8	Mean value	0, 0452039	0,0191778
	Dispersion	0,0143489	0,00654695
	Asymmetry	0,00358164	0,00041628
	Excess	0,00116815	0,000168363

Source: Own calculations

Expected value can be used most simple way for interpretation. It can be transformed to the tradition points or grade marks via linguistic interpretation, depending on range of rating grade, Dispersion characterizes coherence of opinions of experts.

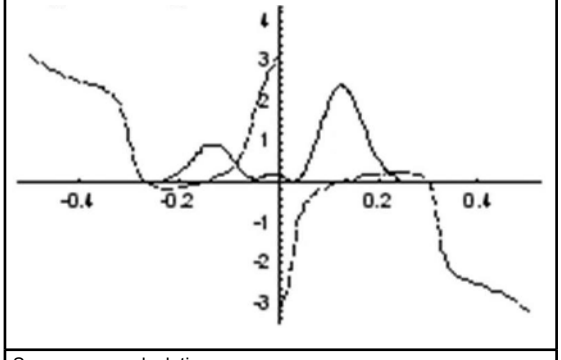
$$\sigma^2 = \int_{-} (r-m)^2\,|\psi(r)|^2\,dr \qquad (7)$$

Figure 3a: The same as in Fig.2a but for i=8. Solid curve is for modulus, dashing curve is for argument input competence set, b) output competence set.



Source: own calculations

Figure 3b: The same as in Fig.2b but for i=8. Solid curve is for modulus, dashing curve is for argument output competence set.



Source: own calculations

In our case when only one expert evaluates different competences the dispersion demonstrate the difference in evaluation of different competences. It depends on previous history of trainee and can be used as additional parameter to the mean value.

The asymmetry is defines as usual:

$$\gamma_1 = \frac{\int_{-} (r-m)^3\,|\psi(r)|^2\,dr}{\left(\sigma_k^2\right)^{\frac{3}{2}}} \qquad (8)$$

The negative value of asymmetry demonstrates dominant impact of negative deviations to mean value. If this quantity becomes negative after class it means that some management action is necessary.

The excess

$$\gamma_1 = \frac{\int_{-} (r-m)^3\,|\psi(r)|^2\,dr}{\left(\sigma_k^2\right)^{\frac{3}{2}}} \qquad (9)$$

also can be used to compare the resulting distribution with Gaussian (which has excess is equal to 0). These characteristics (6-8) can be used for description of trainee characteristics before and after class.

Moreover it can be used for evaluation of education process effectiveness [13]. For the estimation of effectiveness we must compare the resulting SF before and after education

process. The most effective case is: trainee starts from low grade and finish with high grade. Effect is zero if low (high) level remains low(high) after class.

For quantitative comparison we propose to use function which is similar to transfer function or frequency response function in physics and mathematics. As usually this function is determined as

$$W(p) = \frac{Y(p)}{X(p)}, \qquad (10)$$

where is *Y(p)* and *X(p)* are Fourier transformation of input and output SFs at interval [-0.5,0.5]. We can use also series in terms of moments:

$$Y(p) = \int \mu_i(r) e^{ipr} dr = \int \mu_i(r) \left(1 + ipr + \frac{(ip)^2}{2} r^2 + \frac{(ip)^3}{6} r^3 + \frac{(ip)^4}{24} r^4 + \frac{(ip)^5}{120} r^5 + \ldots\right) dr / \int \mu_i(r) dr,$$

$$X(p) = \int \mu_f(r) e^{ipr} dr = \int \mu_f(r) \left(1 + ipr + \frac{(ip)^2}{2} r^2 + \frac{(ip)^3}{6} r^3 + \frac{(ip)^4}{24} r^4 + \frac{(ip)^5}{120} r^5 + \ldots\right) dr / \int \mu_f(r) dr,$$

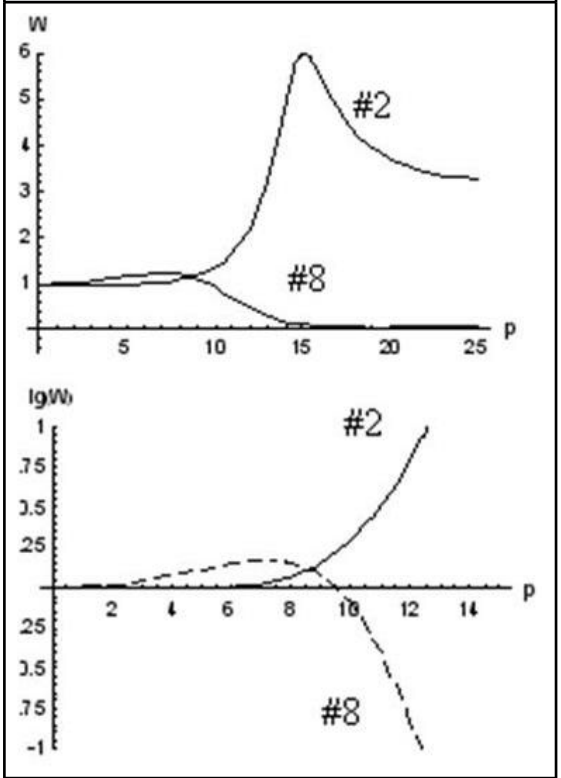
where $\mu_{i,f}(r)$ is square modulus of resulting SF(*i* stands for initial, *f* stands for final state). This result demonstrates the relation of moments with characteristic function. To make difference more pronounced log-scale can be used:

$$W_l(p) = \ln\left[\frac{Y(p)}{X(p)}\right],$$

,thus we should plot *w(p)*=Re[$W_l(p)$]. When education process has no effect, *w(p)=0*. Positive effect occurs at *w(p)>0*, negative at *w(p)<0*. It should be noted that w(0)=0 in correspondence with definition (10). "Frequency" p does not connected with real temporal scale, but reflects the structure of SF. If we use discrete Fourier transformation at interval [-0.5,0.5] we have "frequency" step $2\pi/N$, where *N* is the number of discrete points at "temporal" interval of variation of *r*. Evidently, the sign of *w* at moderate values of *p* mostly determines by mean values before and after class. That coincides with intuitive linguistic term: increase of rating grade means effective education. This is not so obvious when higher moments become important.

In Fig.4 the results of calculation of transfer function are shown for the same students #2 and #8. It is easy to see that till "frequency" about 9 there is only small difference between them. However, for larger values of "frequency" the difference becomes very large, changing sign in log-scale.

Figure 4: Transfer functions (frequency response characteristics) of square modulus of status function of students #2 and #8



Source: own calculations

Conclusion

In this paper we introduce new functions which are generalization of well-known membership functions in fuzzy set theory. These functions were named as status functions. They contain real and imaginary parts. Its can be treats as analog of wave functions in quantum mechanics. Rating procedure with these function applied looks similar to common procedure of evaluation using MF in fuzzy method, however in our case we should calculate square of modulus of these functions and interpreter it as probability density of rating variable, then apply some procedure to transform it to linguistic terms. The use of complex function leads to introducing personal characteristic grades of trainee similar to momentum in quantum mechanics. It give possibilities to consider status functions similar to state vectors and to formulate a problem to derive the "dynamical equations" for these "state vectors" similarly to Schrödinger equation in quantum mechanics.

The competence evaluation (rating) can be made using the expert evaluation of GK and personal properties and calculation of superposition of function similar to MF, overlapping but orthogonal and normalized to 1. GK are represented as weights of these basic SF, while phasor factors are constructed from evaluation of personal characteristics. Resulting SF give resulting grade depending on square of modulus of this function shape treated as probability distribution function. It gives mean value as resulting grade. Other moments of this probability distribution gives additional information, useful for management of the education process. The phase of resulting SF can be used for predicting of further results of education for given set of competence. The results of experiment are presented, demonstrating the ability of the method.

To estimate the effectiveness of education process we propose to use transfer function or frequency response function. It can be calculated as discrete Fourier transform at the interval of rating variable.

For simplicity, we use only "1D rating" in which only one grade scale was used. Large number of competence and characteristics were combined to obtain total resulting grade. Multi-dimensional evaluation method now is in progress.

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